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MATHEMATICS
4016/02
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1 (a) (i)
$$MB = \sqrt{(DB)^2 - (DM)^2}$$
$$= \sqrt{72^2 - 43^2}$$
$$= \sqrt{3335}$$
$$= 57.749$$
$$\approx 57.7 \text{ m}$$

(ii)
$$AB = AM + MB$$
$$= 57.749 + \frac{43}{\tan 62°}$$
$$= 80.613$$
$$\approx 80.6 \text{ m}$$

(iii)
$$CD = \frac{72}{\sin 23°}$$
$$= 184.27$$
$$\approx 184 \text{ m}$$

(b)
$$M\hat{B}D = \sin^{-1}\frac{43}{72}$$
$$= 36.671°$$
Bearing of D from $B = 270° - 36.671°$
$$= 233.329°$$
$$\approx 233.3°$$

2 (a)
$$\frac{x}{8} = \frac{50}{x}$$
$$x^2 = 8 \times 50 = 400$$
$$x = \pm\sqrt{400}$$
$$= \pm 20$$

(b)
$$\frac{t+p}{4} = \frac{q}{5}$$
$$t + p = \frac{4q}{5}$$
$$t = \frac{4q}{5} - p$$
$$\therefore t = \frac{4q - 5p}{5}$$

(c) (i)
$$y = a + \frac{600}{x}$$
$$17 = a + \frac{600}{50}$$
$$\therefore a = 17 - 12 = 5$$

(ii)
$$y = 5 + \frac{600}{x}$$
$$= 5 + \frac{600}{100}$$
$$= \$11$$

(iii)
$$\text{Total cost} = 300\left(5 + \frac{600}{300}\right)$$
$$= 300(5 + 2)$$
$$= \$2100$$

(iv)
$$y = 5 + \frac{600}{x}$$
$$5.2 = 5 + \frac{600}{x}$$
$$0.2 = \frac{600}{x}$$
$$x = \frac{600}{0.2} = 3000 \text{ copies}$$

3 (a) (i)
$$\text{Total amount} = 1299 \times \frac{1}{3} + 24 \times 40.30$$
$$= \$1400.20$$

(ii)
$$\frac{1400.20 - 1299}{1299} \times 100\% = 7\frac{1027}{1299}\%$$
$$\approx 7.79\%$$

(b)
$$1299(1.06^3 - 1) = \$248.13$$

(c)
$$759 \times \frac{100}{115} = \$660$$

4 (a)
$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{4}{3}(x - (-5))$$
$$3y = 4x + 4(5) + 3(4)$$
$$3y = 4x + 32$$
The equation of line AB is $3y = 4x + 32$.

(b)
$$2x + 9y = 68$$
$$2x = 68 - 9y$$
$$4x = 136 - 18y$$
Substitute this equation into $3y = 4x + 32$.
$$3y = 136 - 18y + 32$$
$$21y = 168$$
$$y = 8$$
$$x = \frac{68 - 9(8)}{2} = -2$$
The coordinates of B is $(-2, 8)$.

(c) (i)
$$|\overrightarrow{AE}| = \sqrt{6^2 + 1^2} = \sqrt{37} \approx 6.08$$

(ii)
$$E(-5 + 6, 4 + 1) \Rightarrow E(1, 5)$$

(iii)
$$\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD} = \begin{pmatrix}1\\5\end{pmatrix} - \begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}-3\\3\end{pmatrix}$$
$$\overrightarrow{DB} = \overrightarrow{OB} - \overrightarrow{OD} = \begin{pmatrix}-2\\8\end{pmatrix} - \begin{pmatrix}4\\2\end{pmatrix} = \begin{pmatrix}-6\\6\end{pmatrix}$$

(iv)
1. D, E and B are collinear since there is a common vector $\begin{pmatrix}-1\\1\end{pmatrix}$.
2. E is exactly midway between B and D since $\overrightarrow{DB} = 2\overrightarrow{DE}$.

5 **(a)** **(i)** $X\hat{C}D = \dfrac{360°}{15} = 24°$

(ii) $C\hat{X}D = 180° - 2(24°) = 132°$

(b) Since $X\hat{D}C = 24°$, triangle XCD is isosceles.

$\Rightarrow XC = XD$

Given $BC = DE$,

$XB = XC + BC = XD + DE = XE$

(c) $B\hat{E}D = C\hat{D}X = 24°$

Let Y denote a point on FE extended.

$D\hat{E}Y = 24°$ (exterior angle)

$B\hat{E}F = 180° - 24° - 24° = 132°$

6 **(a)** Number of hours taken $= \dfrac{42}{x}$

(b) Number of hours taken $= \dfrac{42}{x - \frac{1}{2}}$

(c) $$\frac{42}{x-\frac{1}{2}} - \frac{42}{x} = \frac{10}{60}$$

$$6\left[42x - 42(x - \tfrac{1}{2})\right] = x(x - \tfrac{1}{2})$$

$$252x - 252x + 126 = x^2 - \frac{x}{2}$$

$$2x^2 - x - 252 = 0$$

(d) $2x^2 - x - 252 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-252)}}{2(2)}$$

$x = 11.478$ OR $x = -10.978$

(e) Since x is positive, let $x = 11.478$.

$$\text{Time taken} = \frac{42}{11.478}$$

$= 3.659$ hours

≈ 3 h 39 min 33 s

7 **(a)** $170^2 = 95^2 + 102^2 - 2(95)(102)\cos P\hat{Q}R$

$$P\hat{Q}R = \cos^{-1}\left(\frac{170^2 - 95^2 - 102^2}{-2(95)(102)}\right)$$

$= 119.255°$

$\approx 119.3°$

(b) Angle of depression of Q from B

$= \tan^{-1}\dfrac{23}{95}$

$= 13.6097°$

$\approx 13.6°$

(c) $\dfrac{1}{2}(170)(RS)\sin 52° = 5200$

$$RS = \frac{2(5200)}{170\sin 52°}$$

$= 77.634$

≈ 77.6 m

(d) **(i)** $77.634 \div 3 = 25.878$

$\therefore$ 26 panels need to be bought

(ii) Number of posts required

$= 26 + 1$

$= 27$

Total cost $= 26(28.50) + 27(14.95)$

$= \$1144.65$

8 **(a)** **(i)** $PRQ = 44 - 2(8) = 28$

From $s = r\theta$,

$28 = 8\theta$

$\theta = 3\dfrac{1}{2}$

(ii) $P\hat{O}Q = 2\pi - 3\dfrac{1}{2}$

Area of triangle POQ

$= \dfrac{1}{2}(8)(8)\sin P\hat{O}Q$

$= 11.225$

≈ 11.2 m^2

(iii) Total area $= 11.225 + \dfrac{1}{2}r^2\theta$

$= 11.225 + \dfrac{1}{2}(8^2)(3.5)$

$= 11.225 + 112$

≈ 123 m^2

(b) **(i)** Total volume

= Volume of pyramid

+Volume of cuboid

$= \dfrac{1}{3} \times 10 \times 10 \times 12 + 30 \times 10 \times 10$

$= 3400$ cm^3

(ii) Let M denote the midpoint of AB.

$MN = 10 \div 2 = 5$

$VM = \sqrt{(MN)^2 + (VN)^2}$

$= \sqrt{5^2 + 12^2}$

$= 13$

Area of triangle $VAB = \dfrac{1}{2} \times 10 \times 13$

$= 65$ cm^2

Total area $= 4(65) + 4(30 \times 10)$

$= 1460$ cm^2

9 **(a)** See graph below

(b) **(i)** $m = 5.15$

(ii) $t = 17$

(iii) $t = 31$

(c) **(i)** $$\text{Gradient} = \frac{3.55 - 2.20}{-10 - 40}$$

$$= \frac{1.35}{-50}$$

$= -0.027$

(ii) The gradient represents the rate of change of mass at the particular instant. When $t = 7$, the mass is decreasing at –0.027 kg/day instantaneously.

(d) The relationship between m and t may be different for values of t beyond the given range. In this case, $t = 365 > 70$.

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10 (a) (i)

$a = 28 \div 4 = 7$

$b = 60 - (12 + 15 + 10 + 7 + 4 + 0 + 2 + 1) = 9$

$c = 12 \times 0 = 0$

$d = 3 \times 9 = 27$

$e = 0 + 15 + 20 + 27 + 28 + 20 + 0 + 14 + 8$

$= 132$

(ii)

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{132}{60} = 2\frac{1}{5}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{510}{60} - \left(2\frac{1}{5}\right)^2}$$

$$= \sqrt{\frac{183}{50}}$$

$$= 1.91$$

(b) Probability = 0

(c) Number of pupils who had read more than 4 books

$= 4 + 2 + 1 = 7$

P(both had read more than 4 books)

$$= \frac{7}{60} \times \frac{6}{59} = \frac{7}{590}$$

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9 (a)