

1 $3 \cot^2 \theta + 10 \operatorname{cosec} \theta = 5$

$$3(\operatorname{cosec}^2 \theta - 1) + 10 \operatorname{cosec} \theta - 5 = 0$$

$$3 \operatorname{cosec}^2 \theta - 3 + 10 \operatorname{cosec} \theta - 5 = 0$$

$$\frac{3}{\sin^2 \theta} - 8 + \frac{10}{\sin \theta} = 0$$

$$3 - 8 \sin^2 \theta + 10 \sin \theta = 0$$

$$8 \sin^2 \theta - 10 \sin \theta - 3 = 0$$

$$(2 \sin \theta - 3)(4 \sin \theta + 1) = 0$$

$$2 \sin \theta = 3 \quad \text{OR} \quad 4 \sin \theta = -1$$

$$\sin \theta = \frac{3}{2} \text{ (rej.)} \quad \sin \theta = -\frac{1}{4}$$

$$\text{Basic angle } \theta = \sin^{-1} \frac{1}{4} = 14.478^\circ$$

$$\theta = 180^\circ + 14.478^\circ \quad \text{OR} \quad \theta = 360^\circ - 14.478^\circ$$

$$\approx 194.5^\circ \quad \approx 345.5^\circ$$

2 (i) Considering similar triangles PQR and PWW ,

$$\frac{PQ}{QR} = \frac{PW}{WW}$$

$$\frac{8}{12} = \frac{8-y}{x}$$

$$\frac{8x}{12} = 8-y$$

$$y = 8 - \frac{2x}{3}$$

(ii) $A = xy$

$$= x \left(8 - \frac{2x}{3} \right)$$

$$= 8x - \frac{2x^2}{3}$$

(iii) $\frac{dA}{dx} = 8 - \frac{4x}{3} = 0$

$$8 = \frac{4x}{3}$$

$$(8)(3) = 4x$$

$$x = \frac{24}{4} = 6$$

$$\frac{d^2A}{dx^2} = -\frac{4}{3} < 0$$

$$\text{When } x = 6, A = 8(6) - \frac{2(6)^2}{3} = 24.$$

Since $\frac{d^2A}{dx^2} < 0$, the maximum value of A is 24.

3 $32^x \times 2^y = 1$ $3^{x-12} \div 27^y = 81^{\frac{1}{x}}$
 $2^{5x} \times 2^y = 2^0$ $3^{x-12} \div 3^{3y} = 3^{\frac{4}{x}}$
 $2^{5x+y} = 2^0$ $3^{x-12-3y} = 3^{\frac{4}{x}}$
 $5x + y = 0$ $x - 12 - 3y = \frac{4}{x}$
 $y = -5x$ $x^2 - 12x - 3xy = 4$
 $x^2 - 12x - 3x(-5x) = 4$

$$x^2 - 12x + 15x^2 - 4 = 0$$

$$16x^2 - 12x - 4 = 0$$

$$4x^2 - 3x - 1 = 0$$

$$(4x+1)(x-1) = 0$$

$$x = -\frac{1}{4} \quad \text{OR} \quad x = 1$$

$$y = -5\left(-\frac{1}{4}\right) = -5(1) = -5$$

$$y = -5\left(-\frac{1}{4}\right) = -5$$

The solutions are $x = -\frac{1}{4}$, $y = \frac{5}{4}$ and

$x = 1$, $y = -5$.

4 (i) Constant term in $\left(x - \frac{k}{x^3}\right)^8 = \binom{8}{2} x^6 \left(-\frac{k}{x^3}\right)^2$
 $= 28x^6 \left(\frac{k^2}{x^6}\right)$
 $= 28k^2 = 7$

$$k^2 = \frac{1}{4} \Rightarrow k = \pm \frac{1}{2}$$

Since k is positive, $k = \frac{1}{2}$.

(ii) x^{-4} term in $\left(x - \frac{k}{x^3}\right)^8 = \binom{8}{3} x^5 \left(-\frac{1}{2x^3}\right)^3$
 $= 56x^5 \left(-\frac{1}{8x^9}\right) = -\frac{7}{x^4}$

$$\text{Constant term in } (1+x^4) \left(x - \frac{1}{2x^3}\right)^8$$

$$= \text{Constant in } (1+x^4) \times \text{Constant in } \left(x - \frac{1}{2x^3}\right)^8$$

$$+ x^4 \text{ term in } (1+x^4) \times x^{-4} \text{ term in } \left(x - \frac{1}{2x^3}\right)^8$$

$$= (1)(7) + (x^4) \left(-\frac{7}{x^4}\right)$$

$$= 7 - 7$$

$$= 0$$

Since the constant term is zero, there is no constant term.

5 (i) $y = 5 - e^{2x}$
 When $x = 0$, $y = 5 - e^{2(0)} = 5 - 1 = 4$
 When $y = 0$, $0 = 5 - e^{2x}$
 $5 = e^{2x}$
 $2x = \ln 5$
 $x = \frac{\ln 5}{2}$

$\therefore A(0,4)$ and $B\left(\frac{\ln 5}{2}, 0\right)$

Let P and Q denote the points $(0, k)$ and $(\ln 5, k)$ respectively.

Consider the similar triangles AOB and APQ .

$$\frac{4-0}{\frac{\ln 5}{2}} = \frac{4-k}{\ln 5}$$

$$4 \ln 5 = (4-k) \frac{\ln 5}{2}$$

$$8 = 4 - k$$

$$\therefore k = -4$$

(ii) $x = \ln \sqrt{9-x}$

$$e^x = \sqrt{9-x}$$

$$e^{2x} = 9-x$$

$$5 - e^{2x} = 5 - (9-x)$$

$$= x - 4$$

The equation of the straight line is $y = x - 4$.

6 (i) Let k denote $B\hat{X}A$.

$$Y\hat{X}A = B\hat{X}A = k \quad (\text{given } XA \text{ bisects } Y\hat{X}B)$$

$$A\hat{C}X = Y\hat{X}A = k \quad (\text{tangent chord theorem})$$

$$A\hat{B}X = A\hat{C}X = k \quad (\text{angle in the same segment})$$

Since $B\hat{X}A = A\hat{B}X = k$, ABX is an isosceles triangle. $\therefore AX = AB$

(ii) $A\hat{C}B = A\hat{X}B = k$ (angle in the same segment)

Since $A\hat{C}X = A\hat{C}B = k$, and D lies on AC , CD bisects $X\hat{C}B$.

(iii) In triangles CDX and CBA ,

$$X\hat{C}D = A\hat{C}B \quad (\text{since } CD \text{ bisects } X\hat{C}B)$$

$$C\hat{X}D = C\hat{A}B \quad (\text{angle in the same segment})$$

$$C\hat{D}X = C\hat{B}A \quad (\text{angle sum of a triangle})$$

By the AAA property, triangles CDX and CBA are similar.

7 (i) $y = \sqrt{2x+5}$
 $\frac{dy}{dx} = \frac{2}{2\sqrt{2x+5}}$
 $= \frac{1}{\sqrt{2x+5}}$

When $x = 2$, $\frac{dy}{dx} = \frac{1}{\sqrt{2(2)+5}} = \frac{1}{3}$

$$x\text{-coordinate of } Q = -\frac{5}{2} \Rightarrow Q\left(-\frac{5}{2}, 0\right)$$

$$y - 0 = \frac{1}{3}\left(x - \left(-\frac{5}{2}\right)\right) = \frac{1}{3}x + \frac{5}{6}$$

The equation of QR is $y = \frac{1}{3}x + \frac{5}{6}$.

(ii) Area of QPS

$$= \int_{-\frac{5}{2}}^2 \sqrt{2x+5} \, dx$$

$$= \left[\frac{\sqrt{(2x+5)^3}}{2 \times \frac{3}{2}} \right]_{-\frac{5}{2}}^2$$

$$= \frac{\sqrt{(2(2)+5)^3}}{2 \times \frac{3}{2}} - \frac{\sqrt{(2(-\frac{5}{2})+5)^3}}{2 \times \frac{3}{2}}$$

$$= 9 \text{ square units}$$

Considering QR , when $x = 2$,

$$y = \frac{1}{3}(2) + \frac{5}{6} = \frac{3}{2}$$

Area of QST = Area of QSR

$$= \frac{1}{2}\left(2 - \left(-\frac{5}{2}\right)\right)\left(\frac{3}{2}\right)$$

$$= \frac{27}{8} \text{ square units}$$

$$\text{Total area} = 9 + \frac{27}{8} = \frac{99}{8} \text{ square units}$$

8 (i) $v_P = \int a_P \, dt$ OR $v_Q = \int a_Q \, dt$
 $= \int 1.5 \, dt$ $= \int 1 + \frac{t}{2} \, dt$
 $= 1.5t + c$ $= t + \frac{t^2}{2(2)} + c$
 $= t + \frac{t^2}{4} + c$

When $t = 0$, $c = 9$

$$v_P = 1.5t + 9$$

When $t = 0$, $c = 0$

$$v_Q = t + \frac{t^2}{4}$$

(ii) $s_P = \int v_P \, dt$ $s_Q = \int v_Q \, dt$
 $= \int 9 + 1.5t \, dt$ $= \int t + \frac{t^2}{4} \, dt$
 $= 9t + \frac{1.5t^2}{2} + c$ $= \frac{t^2}{2} + \frac{t^3}{12} + d$
 $= 9t + \frac{3t^2}{4} + c$

Since both particles started at the origin, when $t = 0$, $s_P = c = 0$ and $s_Q = d = 0$.

The distance travelled by P is $s_P = 9t + \frac{3t^2}{4}$.

The distance travelled by Q is $s_Q = \frac{t^2}{2} + \frac{t^3}{12}$.

(iii) $s_P = s_Q$
 $9t + \frac{3t^2}{4} = \frac{t^2}{2} + \frac{t^3}{12}$
 $108t + 9t^2 = 6t^2 + t^3$
 $t^3 - 3t^2 - 108t = 0$
 $t(t^2 - 3t - 108) = 0$
 $t(t+9)(t-12) = 0$
 $t = 0, t = -9$ OR $t = 12$
 $s = 9t + \frac{3t^2}{4}$
 $= 216$ m

(iv) $v_P = 9 + 1.5t = 27$ m/s
 $v_Q = t + \frac{t^2}{4} = 48$ m/s

9 (i) $AB = BC$
 $\sqrt{(8-0)^2 + (14-5)^2} = \sqrt{(k-8)^2 + (15-14)^2}$
 $145 = k^2 - 16k + 64 + 1$
 $k^2 - 16k - 80 = 0$
 $(k-20)(k+4) = 0$
 $k = 20$ OR $k = -4$

Since k is positive, we have $k = 20$.

(ii) Since $AD = CD$, D lies on the perpendicular bisector of AC .

Let M denote the midpoint of AC .

$$M\left(\frac{0+20}{2}, \frac{5+15}{2}\right) \Rightarrow M(10, 10)$$

$$\text{Gradient of } BM = \frac{14-10}{8-10} = \frac{4}{-2} = -2$$

$$y - 10 = -2(x - 10)$$

$$y = -2x + 20 + 10$$

$$= -2x + 30$$

The equation of the line BD is $y = -2x + 10$.

To find the coordinates of D , let $y = 0$.

$$0 = -2x + 30$$

$$2x = 30 \Rightarrow x = 15$$

$\therefore D(15, 0)$

(iii) $BM = \sqrt{(8-10)^2 + (14-10)^2} = \sqrt{20} = 2\sqrt{5}$
 $MD = \sqrt{(10-15)^2 + (10-0)^2} = \sqrt{125} = 5\sqrt{5}$

$$\frac{\text{Area of } \triangle ABC}{\text{Area of quadrilateral } ABCD}$$

$$= \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ABC + \text{Area of } \triangle ADC}$$

$$= \frac{\frac{1}{2}(AC)(BM)}{\frac{1}{2}(AC)(BM) + \frac{1}{2}(AC)(MD)}$$

$$= \frac{\frac{1}{2}(AC)(2\sqrt{5})}{\frac{1}{2}(AC)(2\sqrt{5}) + \frac{1}{2}(AC)(5\sqrt{5})}$$

$$= \frac{\frac{1}{2}(AC)(2\sqrt{5})}{\frac{1}{2}(AC)(7\sqrt{5})}$$

$$= \frac{2}{7}$$

\therefore Area of $\triangle ABC$
 $= \frac{2}{7} \times \text{Area of quadrilateral } ABCD$

10 (i) $\frac{3x^2 + 4x - 20}{(2x+1)(x^2+4)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+4}$
 $3x^2 + 4x - 20 = A(x^2+4) + (Bx+C)(2x+1)$

Let $x = -0.5$

$$3(-0.5)^2 + 4(-0.5) - 20 = A((-0.5)^2 + 4) + 0$$

$$-\frac{85}{4} = \frac{17}{4}A$$

$$A = -\frac{85}{4} \div \frac{17}{4} = -5$$

$$\Rightarrow 3x^2 + 4x - 20 = -5(x^2 + 4) + (Bx + C)(2x + 1)$$

Let $x = 0$

$$-20 = -5(4) + 3C$$

$$3C = -5(4) + 20 = 0$$

$$C = 0$$

$$\Rightarrow 3x^2 + 4x - 20 = -5(x^2 + 4) + (Bx)(2x + 1)$$

Let $x = 1$

$$3(1)^2 + 4(1) - 20 = -5((1)^2 + 4) + (B(1))(2(1) + 1)$$

$$-13 = -25 + 3B$$

$$3B = -13 + 25 = 12$$

$$B = 4$$

$$\therefore \frac{3x^2 + 4x - 20}{(2x+1)(x^2+4)} = \frac{-5}{2x+1} + \frac{4x}{x^2+4}, \text{ where}$$

$$A = -5, B = 4, C = 0.$$

(ii) $\frac{d}{dx} \ln(x^2 + 4)$
 $= \frac{1}{x^2 + 4} \times 2x$
 $= \frac{2x}{x^2 + 4}$

$$\begin{aligned}
 \text{(iii)} \quad & \int \frac{3x^2 + 4x - 20}{(2x+1)(x^2+4)} dx \\
 &= \int \frac{-5}{2x+1} + \frac{4x}{x^2+4} dx \\
 &= \int \frac{-5}{2x+1} dx + \int \frac{4x}{x^2+4} dx \\
 &= -5 \int \frac{1}{2x+1} dx + 2 \int \frac{2x}{x^2+4} dx \\
 &= -5 \frac{\ln(2x+1)}{2} + 2 \ln(x^2+4) + c \\
 &= -\frac{5}{2} \ln(2x+1) + 2 \ln(x^2+4) + c, \quad x > -\frac{1}{2}
 \end{aligned}$$

$$\text{11 (i)} \quad A\left(\frac{\pi}{2}, 5\right) \quad B\left(\frac{3\pi}{2}, -1\right) \quad C(\pi, -4)$$

$$\text{(ii)} \quad 4 \cos x = 2 + 3 \sin x$$

$$4 \cos x - 3 \sin x = 2$$

$$R \cos(x + \alpha) = 2$$

$$R = \sqrt{4^2 + 3^2} = 5$$

$$\alpha = \tan^{-1} \frac{b}{a} = \tan^{-1} \frac{3}{4} = 0.64350 \approx 0.644$$

$$\text{Now, } 5 \cos(x + 0.644) = 2$$

$$\cos(x + 0.644) = 0.4$$

$$\text{where } \alpha = 0.644, k = 0.4.$$

$$\text{(iii)} \quad \cos(x + 0.644) = 0.4$$

$$\text{Basic angle } (x + 0.644) = \cos^{-1} 0.4$$

$$= 1.1593$$

$$x + 0.644 = 1.1593 \quad \text{OR} \quad x + 0.644 = 2\pi - 1.1593$$

$$x \approx 0.516$$

$$x \approx 4.48$$

$$x\text{-coordinate of } D = 0.516$$

$$x\text{-coordinate of } E = 4.48$$
