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ADDITIONAL MATHEMATICS

4038/01

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1 (i)

$f(x) = x^4 - x^3 + kx - 4$

$f(2) = 2^4 - 2^3 + 2k - 4 = 0$

$16 - 8 + 2k - 4 = 0$

$2k = -4$

$k = -2$

(ii)

$f(x) = x^4 - x^3 - 2x - 4$

$f(-2) = (-2)^4 - (-2)^3 - 2(-2) - 4$

$= 16 + 8 + 4 - 4$

$= 24$

The remainder is 24.

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2 (i)

$\text{LHS} = (\sin x + \cos x)^2$

$= \sin^2 x + 2\sin x\cos x + \cos^2 x$

$= (\sin^2 x + \cos^2 x) + \sin 2x$

$= 1 + \sin 2x = \text{RHS}$

(ii)

$\int_0^{\frac{\pi}{2}} (\sin x + \cos x)^2 \, dx$

$= \int_0^{\frac{\pi}{2}} 1 + \sin 2x \, dx$

$= \left[x - \dfrac{\cos 2x}{2} \right]_0^{\frac{\pi}{2}}$

$= \left(\dfrac{\pi}{2} - \dfrac{\cos 2(\frac{\pi}{2})}{2} \right) - \left(0 - \dfrac{\cos 2(0)}{2} \right)$

$= \dfrac{\pi}{2} - \dfrac{-1}{2} - 0 + \dfrac{1}{2}$

$= \dfrac{\pi}{2} + 1$

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3 (i)

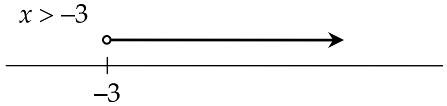
$3(2 - x) < x + 18$

$6 - 3x < x + 18$

$6 - 18 < x + 3x$

$-12 < 4x$

$x > -3$



(ii)

$3(x^2 - 5) > x - 1$

$3x^2 - 15 > x - 1$

$3x^2 - x - 15 + 1 > 0$

$3x^2 - x - 14 > 0$

$(x + 2)(3x - 7) > 0$

$(x + 2) < 0$ OR $(3x - 7) > 0$

$x < -2$ $\quad$ $3x > 7$

$x > \dfrac{7}{3}$

The set of values which satisfy both inequalities is

$\{x \in \mathbb{R} : -3 < x < -2\} \cup \{x \in \mathbb{R} : x > \frac{7}{3}\}$.

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4 (i)

$y = \sin 2x - 3\cos x$

$\dfrac{dy}{dx} = 2\cos 2x + 3\sin x$

$= 2\cos 2(\frac{\pi}{6}) + 3\sin(\frac{\pi}{6})$

$= 2\cos\frac{\pi}{3} + 3\sin\frac{\pi}{6}$

$= 2(\frac{1}{2}) + 3(\frac{1}{2})$

$= \dfrac{5}{2}$

(ii)

$\dfrac{dx}{dt} = 0.06$

$\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$

$= \dfrac{5}{2} \times 0.06$

$= 0.15$

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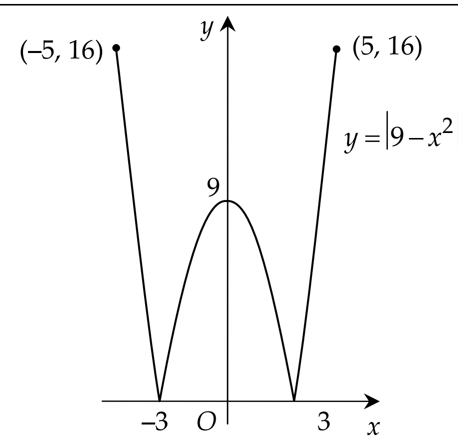
5 (i)



(ii)

$|9 - x^2| = 27$

$9 - x^2 = 27$ OR $9 - x^2 = -27$

$9 - 27 = x^2$ $\quad$ $9 + 27 = x^2$

$-18 = x^2$ $\quad$ $36 = x^2$

$x = \pm 6$

The two x-coordinates are 6 and −6.

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6 (i)

$\dfrac{d}{dx} xe^{2x} = x(2e^{2x}) + e^{2x}(1)$

$= 2xe^{2x} + e^{2x}$

(ii) $\int_0^1 xe^{2x}\ dx$

$= \frac{1}{2}\int_0^1 2xe^{2x} + e^{2x} - e^{2x}\ dx$

$= \frac{1}{2}\left(\int_0^1 2xe^{2x} + e^{2x}\ dx - \int_0^1 e^{2x}\ dx\right)$

$= \frac{1}{2}\left(\left[xe^{2x}\right]_0^1 - \left[\frac{e^{2x}}{2}\right]_0^1\right)$

$= \frac{1}{2}\left(\left[(1)e^{2(1)} + (0)e^{2(0)}\right] - \frac{1}{2}\left[\frac{e^{2(1)}}{2} - \frac{e^{2(0)}}{2}\right]\right)$

$= \frac{1}{2}\left(e^2 - \frac{e^2}{2} + \frac{1}{2}\right)$

$= \frac{1}{2}\left(\frac{e^2+1}{2}\right)$

$= \frac{e^2+1}{4}$

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7 (i)

x	2	8	14	20
$\lg x$	0.301	0.903	1.146	1.301
y	33.00	5.07	2.38	1.47
$\lg y$	1.519	0.705	0.377	0.167

Refer to graph

(ii) $yx^n = k$

$\lg(yx^n) = \lg k$

$\lg y + n\lg x = \lg k$

$\lg y = -n\lg x + \lg k$

Gradient $= -n$

$= \frac{1.66-0.31}{0.2-1.2}$

$= \frac{1.35}{-1}$

$= -1.35$

$n = 1.35$

Vertical intercept $= \lg k$

$= 1.93$

$k = 10^{1.93}$

$= 85.1$

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8 (i) $y = x^3 + 3x^2 - 9x + k$

$\frac{dy}{dx} = 3x^2 + 6x - 9$

$= 3(x^2 + 2x - 3)$

$= 3(x-1)(x+3)$

When y is decreasing, $\frac{dy}{dx} < 0$.

$3(x-1)(x+3) < 0$

$-3 < x < 1$

The set of values of x is $\{x \in \mathbb{R} : -3 < x < 1\}$.

(ii) When the x-axis is tangent to the curve, we have $\frac{dy}{dx} = 0$ and $y = 0$.

$3(x-1)(x+3) = 0$

$x = 1$ OR $x = -3$

At $(1,0)$, $(0) = (1)^3 + 3(1)^2 - 9(1) + k$

$k = -1 - 3 + 9$

$= 5$

At $(-3,0)$, $(0) = (-3)^3 + 3(-3)^2 - 9(-3) + k$

$k = 27 - 27 - 27$

$= -27$

$\therefore k = 5$ OR $k = -27$

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9 For the equation $3x^2 - 2x + 1 = 0$,

$3(x^2 - \frac{2}{3}x + \frac{1}{3}) = 0$

Sum of roots $= \alpha + \beta = \frac{2}{3}$

Product of roots $= \alpha\beta = \frac{1}{3}$

For the new quadratic equation,

Sum of roots $= (2\alpha + \beta) + (\alpha + 2\beta)$

$= 3\alpha + 3\beta$

$= 3(\alpha + \beta)$

$= 3(\frac{2}{3}) = 2$

Product of roots $= (2\alpha + \beta)(\alpha + 2\beta)$

$= 2\alpha^2 + 4\alpha\beta + \alpha\beta + 2\beta^2$

$= 2(\alpha^2 + 2\alpha\beta + \beta^2) + \alpha\beta$

$= 2(\alpha + \beta)^2 + \alpha\beta$

$= 2(\frac{2}{3})^2 + \frac{1}{3} = \frac{11}{9}$

$x^2 - (2)x + (\frac{11}{9}) = 0$

$9x^2 - 18x + 11 = 0$

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10 (i) $\tan 75° = \tan(45° + 30°)$

$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$

$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - (1)\left(\frac{\sqrt{3}}{3}\right)}$

$= \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}}$

$= \frac{3+\sqrt{3}}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$

$= \frac{9 + 6\sqrt{3} + 3}{3^2 - 3}$

$= \frac{12 + 6\sqrt{3}}{3^2 - 3}$

$= 2 + \sqrt{3}$

(ii) $\sec^2 75° = 1 + \tan^2 75°$

$= 1 + (2+\sqrt{3})^2$

$= 1 + (4 + 4\sqrt{3} + 3)$

$= 8 + 4\sqrt{3}$

$= 4(2+\sqrt{3})$

$= 4\tan 75°$

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11 **(i)** $\frac{dy}{dx} = \frac{8}{x^2} - 2$

$y = -\frac{8}{x} - 2x + c$

At $(1,5)$, $5 = -\frac{8}{1} - 2(1) + c$

$c = 5 + 8 + 2$

$= 15$

(ii) $\frac{dy}{dx} = \frac{8}{x^2} - 2 = 0$

$\frac{8}{x^2} = 2$

$8 = 2x^2$

$x^2 = 4$

$x = \pm 2$

The x-coordinates are 2 and –2.

(iii) $\frac{d^2y}{dx^2} = (-2)\frac{8}{x^3}$

$= -\frac{16}{x^3}$

When $x = 2$,

$y = -\frac{8}{2} - 2(2) + 15 = 7$

$\frac{d^2y}{dx^2} = -\frac{16}{2^3} = -2$

Since $\frac{d^2y}{dx^2} < 0$, $(2,7)$ is a maximum point.

When $x = -2$,

$y = -\frac{8}{-2} - 2(-2) + 15 = 23$

$\frac{d^2y}{dx^2} = -\frac{16}{(-2)^3} = 2$

Since $\frac{d^2y}{dx^2} > 0$, $(-2,23)$ is a minimum point.

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12 **(i)** $(x+3)^2 + (y-2)^2 = 5^2$

$(x+3)^2 + (y-2)^2 = 25$

(ii) Given that the circle intersects the y-axis at two points, let $x = 0$.

$(0+3)^2 + (y-2)^2 = 25$

$9 + (y^2 - 4y + 4) = 25$

$y^2 - 4y + 4 + 9 - 25 = 0$

$y^2 - 4y - 12 = 0$

$(y-6)(y+2) = 0$

$y = 6$ OR $y = -2$

$PQ = 6 - (-2) = 8$

(iii) y-coordinate of $B = 2$

(iv) Let B and P be the points $(k,2)$ and $(0,6)$ respectively.

$BP = \sqrt{(k-0)^2 + (2-6)^2}$

$= \sqrt{k^2 + 16}$

$= \sqrt{80}$

$k^2 = 80 - 16 = 64$

$k = \pm 8$

Since the x-coordinate of B is positive, $k = 8$.

The x-coordinate of B is 8.

(v) $(x-8)^2 + (y-2)^2 = (\sqrt{80})^2$

$(x^2 - 16x + 64) + (y^2 - 4y + 4) = 80$

$x^2 + y^2 - 16x - 4y + 64 + 4 - 80 = 0$

$x^2 + y^2 - 16x - 4y - 12 = 0$

$\therefore g = \frac{-16}{2} = -8,\ f = \frac{-4}{2} = -2,\ c = -12$

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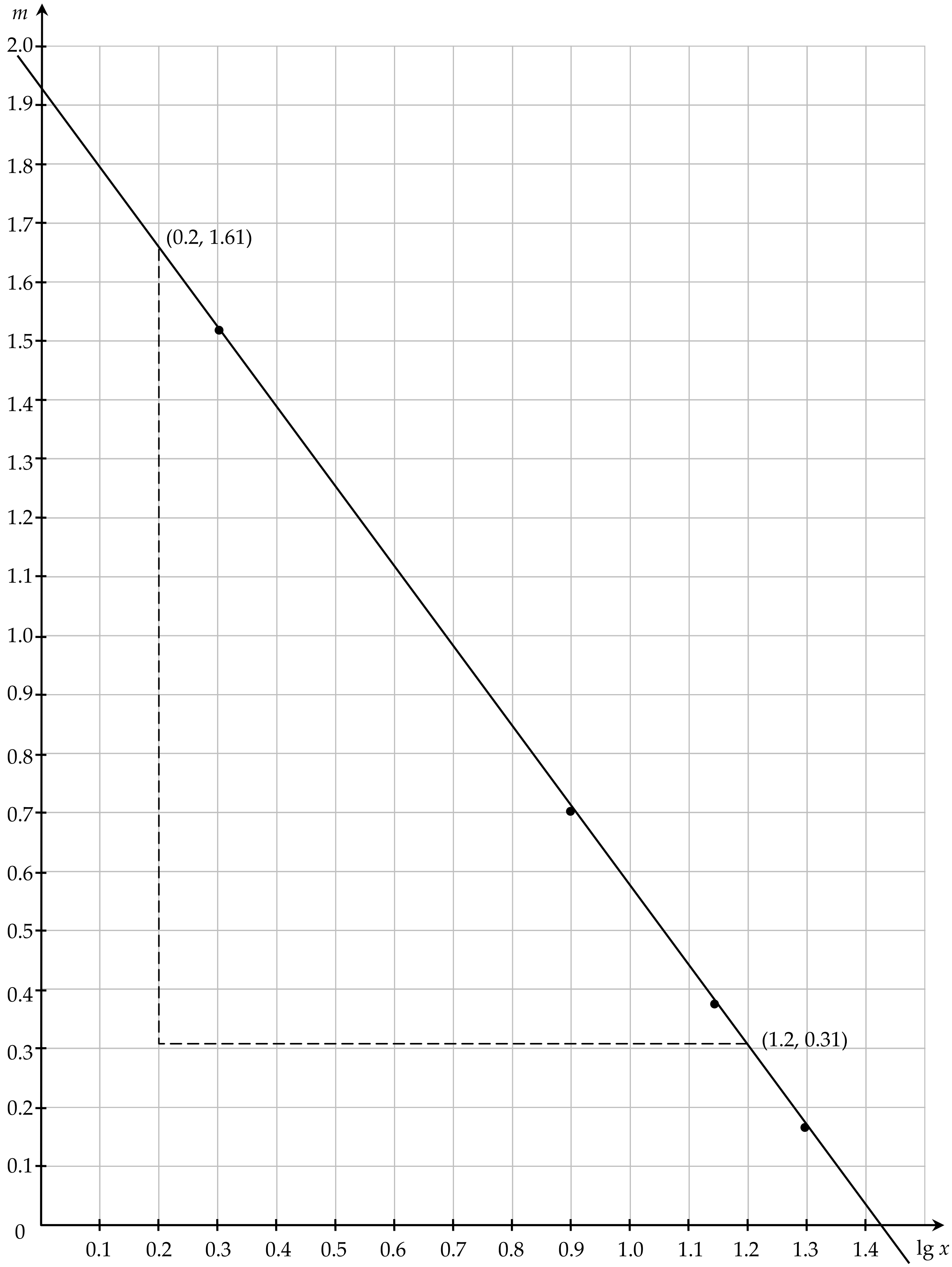
7 (i)



m
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
lg x
(0.2, 1.61)
(1.2, 0.31)